This paper is presented pending completion of the author's specific paper for the UNITE Seminar, Paris 2001.

A NOTE ON THE ECONOMIC INTERPRETATION OF DELAY FUNCTIONS IN ASSIGNMENT PROBLEMS

by

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Presented at the International Symposium on Traffic Equilibrium Methods, Montreal, November 21-23, 1974. The author is grateful to the Ford Motor Company of Canada for supporting this research, to Harold Kuhn and Martin Beckmann for their encouragements, and to Springer-Verlag for publishing it in Lecture Notes in Economics and Mathematical Systems, 118, 368-381, 1976, and transferring the publishing rights back to the author in 2001.

Université de Montréal

Cahier #7421 – Département de sciences économiques, janvier 1975
Publication #16 – Centre de recherche sur les transports, novembre 1974

Retyped with minor corrections, November 2001

Dépôt légal – 1er trimestre 1978

ABSTRACT

The relationship between the time delay curves of traffic assignment problems and supply curves is examined in this paper. A claim is made that most delay curves are simplified cost and/or production functions. The resulting distinction between delay and supply curves is illustrated by differentiating between production and market equilibria in networks, notably in cases in which the former equilibria are observed while the latter are not.

RÉSUMÉ

Cette communication étudie les rapports qui existent entre les courbes de délai (débit-vitesse) utilisées dans les problèmes d'affectation de la circulation et les courbes d'offre. Il y appert que la majorité des fonctions de délai sont des fonctions de coût et/ou des fonctions de production simplifiées. On y est ainsi amené à mieux distinguer les courbes de délai des courbes d'offre en soulignant la différence qui existe, dans un réseau, entre un équilibre dit de production et un équilibre de marché, en particulier lorsque l'équilibre de production est observé sans que l'équilibre de marché le soit.

Introduction[1]

Time delay curves are often identified with supply curves in assignment problems. Used in conjunction with demand curves, they define problems which have the appearance of market problems centered on the existence and stability of equilibrium market clearing solutions. This note purports to explore the relationship between delay and supply curves.

A first section derives delay curves from production and cost functions for the usual travel output along a single street. The second section describes a case in which delay curves might be interpreted as supply curves and lists more realistic cases in which delay and supply curves have so little in common that production equilibria occur on delay curves without supply curves being even observable – due to the presence of stable market disequilibria. The conclusion briefly extends these perspectives to the interpretation of complex network equilibria.

The derivation of delay curves from production and cost functions

A. Travel production and cost functions

Let us assume that a single street goes from i to j and that vehicle trips per hour between those points may be produced with inputs of vehicle-distance per hour and of vehicle-time per hour. The resulting relationship between travel inputs and outputs is represented in figure 1a where OV gives the value of output associated with all values of inputs. OV is a curve and not a surface because, at a given level of output, vehicle-miles cannot be dissociated from vehicle-minutes (there is no factor substitution). Up to the point v, the curve OV is a straight line and exhibits constant returns to scale because vehicles are assumed to travel at the legal speed if they can. Beyond the point v, the production function exhibits rapidly decreasing returns to scale, and factor proportions change.

[1] The author wishes to thank Michael Florian and Sang Nguyen of Université de Montréal for helpful discussions and Harold F. Kuhn of Princeton University and Martin J. Beckmann of Brown University and Technische Universität Munchen for their strong encouragements, notably at a homely dinner, during the famous snowstorm of November 21, 1974.

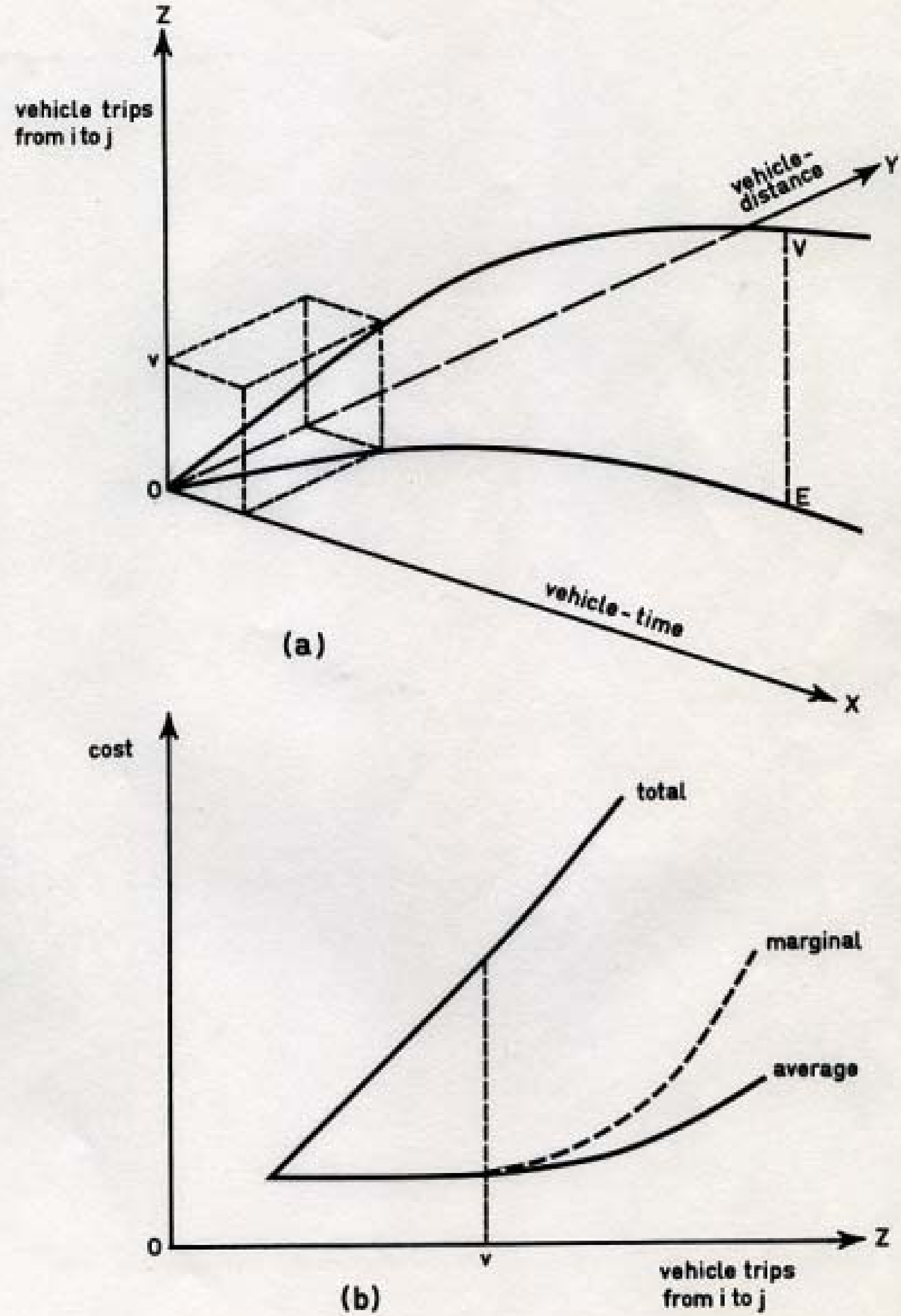
Z
vehicle trips
from i to j
vehicle-
distance
Y
V
v
0
E
vehicle - time
(a)
X
cost
total
marginal
average
0
v
Z
vehicle trips
from i to j
(b)

figure 1

If the prices of the vehicle-distance input (gas + maintenance + depreciation per mile + tolls) and of the vehicle-time input (driver's wage rate + depreciation per time period) are known constants, one may derive the total, average and marginal costs associated with each level of output: these curves are given in figure 1b. The total cost curve gives the minimum total cost for each level of output at given input prices. The "firm's" expansion path is of course simply the curve OE, the projection of OV on the XY input plane; it is independent from variations in relative input prices.

In a complex network, by contrast, the expansion path changes with relative input prices because factor substitution is possible. If there are four itineraries from i to j , as shown in figure 2a, input requirements for v trips per time period vary with relative input prices. As can be seen in figure 2b where the four points are projected on the XY plane, input combination a' minimizes total time but it is the cheapest only if relative input prices are given by a price line such as P_2 rather than by a price line such as P_1 (which embodies a relatively lower price of time than P_2). The cost functions of this network would be step functions because higher trip levels can be produced at equal or greater unit cost.

The computation of cheapest input requirements for each output level is what we may call the <u>network equilibrium production problem</u>. We will not discuss differences between system and user (Potts and Oliver, 1972) optimizing solutions or even ask whether these procedures mimic actual behavior. We will be satisfied with the fact that assignment procedures succeed in computing input requirements which respect the constraints of the production technology and we shall limit our considerations to the single street case for expository purposes.

B. <u>Delay curves as special production/cost curves</u>

The are two ways of obtaining delay functions, both of which involve making the distance input irrelevant.

The first derivation is a transformation of the production function into a one-argument function be neglecting the distance input. This can be effected by projecting the curve OV in

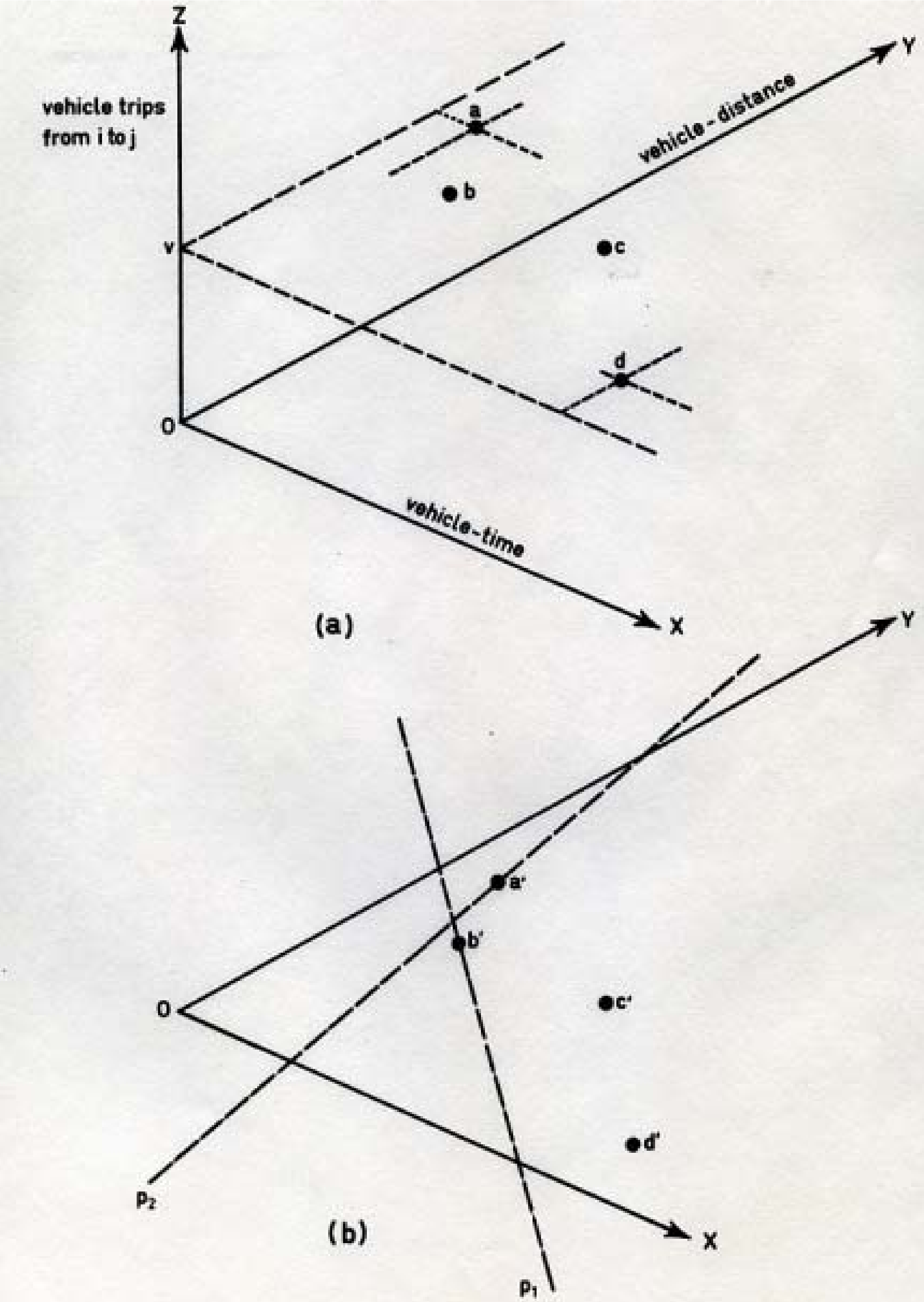
Z
vehicle trips
from i to j
a
b
c
d
v
O
vehicle - distance
Y
vehicle-time
X
(a)
Y
a'
b'
c'
d'
O
p2
(b)
X
p1

figure 2

figure 1a on the XZ plane and by adding the average and marginal curves, as is done in figure 3. The delay curve is the average time curve.

Another interpretation is derived from the assumption of a production function for which the distance input is free and the unit cost of time is one. The resulting time cost curve is identical to that shown in figure 3 except that the vertical axis is labeled "time cost".

Whether delay curves result from simplified production functions or from special cost curves, distance does not matter in the construction of delay curves: it is either not an input, or it is free.

Delay curves and supply curves

If we accept the exceedingly strong assumption that distance is irrelevant, the production equilibrium will certainly be on the delay curve which associates to each output level its minimum average time input requirement. But are these delay curves supply curves?

We need information on market structure to answer that question; let us then examine three classes of markets in which it might be asked. If we need to use figures, we shall adopt the common usage and label the vertical axis in time units; a parallel presentation using the cost label would leave the argument unchanged.

A. Taxi markets

If there are nothing but taxis serving the street in question and if the taxi industry is perfectly competitive, the delay curve is a supply curve: the situation is analogous to the classical case in which each producer is a price taker who has no effective influence on price so that the long-run supply curve for the industry coincides with the average "cost" curve.

As soon as the firm is able to set its own "price", the concept of the supply curve is not strictly applicable any more [2]. A profit maximizing taxi monopoly will choose a particular trip volume such that its marginal (time) revenues and (time) costs are equal. If there is congestion at that point, the chosen point, such as point b in figure 4, is not even on the average delay curve. The delay curve is not the supply curve but simply describes technically feasible choices and/or their costs.

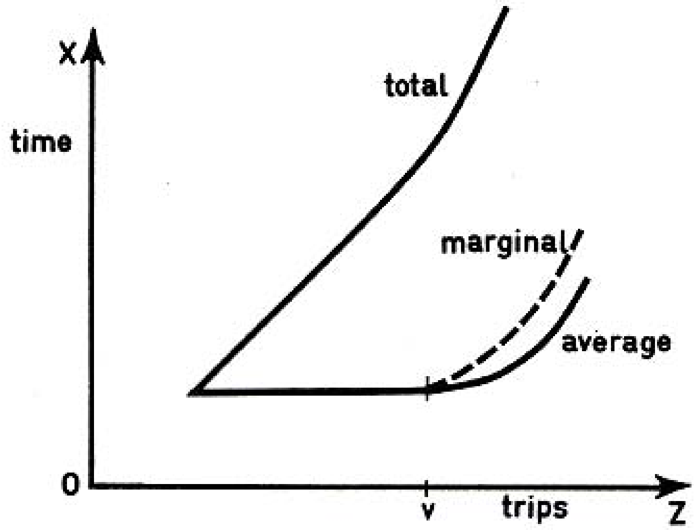

figure 3

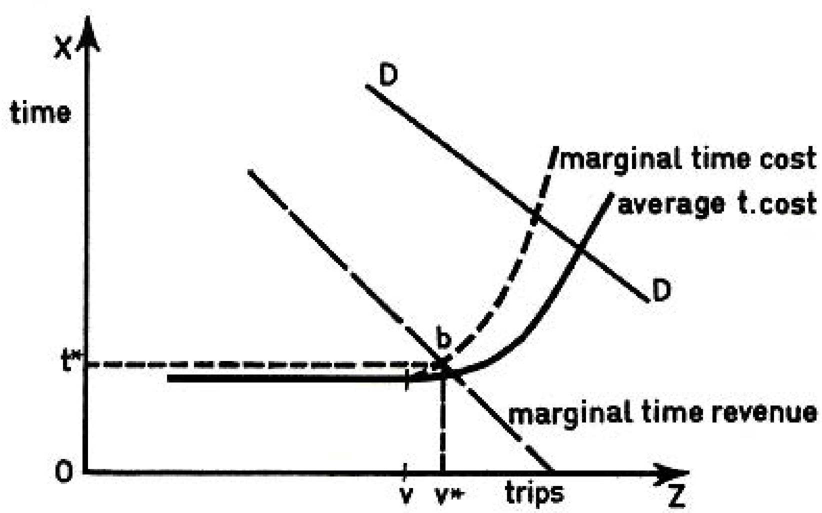

figure 4

B. Private car trip markets

The private car trip market is characterized by the fact that the suppliers are often consumers as well. It is a market in which each driver is a "price" taker; but it is also a market in which consumers do not normally pay the producer. A fair representation of the process might be the following: assume that in each household some members are drivers but all members demand car trips. The suppliers' reaction would be such that they would drive consumers (including themselves) to their destinations more often if the average time required for the trip is small than if it is large: this means that the supply curve - like the demand curve - is downward sloping.

In that market, trips can occur if suppliers are willing to drive and if there are enough consumers who want to travel in the network. Trips will occur if that (time) price is sustainable in the network. Figures 5a and 5b illustrate two likely equilibria. The striped curves give possible outcomes and the delay curves give technically feasible ones; points m and n cannot be maintained because they would imply that drivers purposefully drive slowly in empty streets. The sustainable solutions are those in which the vehicles which do show up are driven as fast as possible. In 5a the resulting values of v and t are on the demand curve; in 5b they are on the supply curve. Both situations are production equilibria but none are market equilibria. Markets would clear if by chance the market equilibrium were identical with the production equilibrium.

This does not ease the task of estimating demand and supply curves but it makes it clear that, even if distance is assumed not to matter, delay curves have little to do with supply curves.

C. Public transit markets

In an urban context, the representative public transit authority will face a trade-off between having buses travel as fast as cars and having them stop for passengers. The chosen travel speed will normally be smaller than the car speed. The optimal speed is chosen among all the points situated above the delay function boundary, for all of these are feasible and perhaps desirable points. A good demand model, a formulation of the authority's objectives and constraints are needed to find appropriate supply points in this considerable domain. In figure 6a the demand curve has not been drawn because it will shift every time the distance between bus stops is

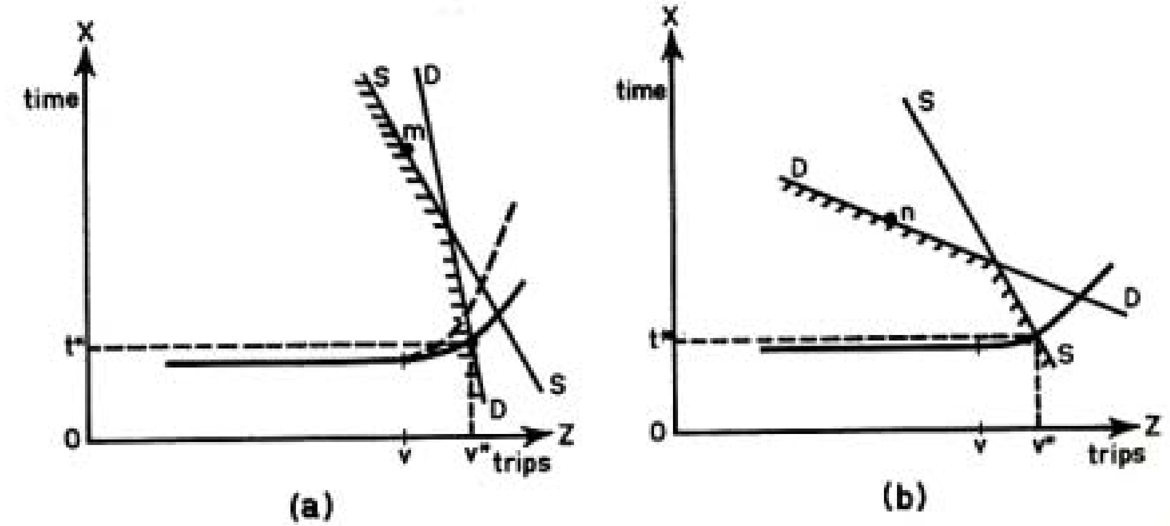

figure 5

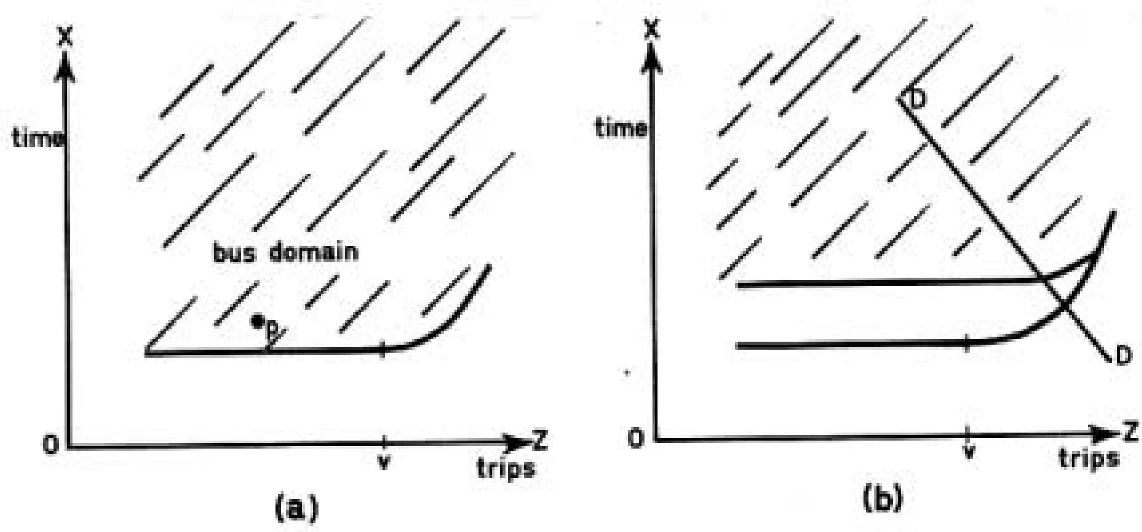

figure 6

changed in order to increase travel speed. In figure 6b a demand curve has been drawn on the assumption that the number of stops along the street is fixed: in consequence the transit authority's area of choice for a supply point has been reduced.

This difficult problem has received little attention in the literature. One can however say that, in this transit context, a time delay curve is nothing more than a boundary: as soon as speed becomes an object of choice, the already tenuous link between delay and supply curves vanishes altogether.

Conclusion

The above perspective distinguishes clearly between production and market equilibria. The examples have been chosen from a single street case for the sake of clarity. In more complicated problems the relevant distinctions are not so easy to make.

If there exists a network between a single given origin-destination pair, the analysis carries over: as in figure 2, one has production processes rather than production functions. Processes combine time and distance inputs (from different streets) in varying proportions and the chosen process depends not only on relative input prices but on control technology: our monopolistic taxi corporation could not assign its cars in a system optimizing fashion without centralized control of individual itineraries. Once the average cost curve is found, the distinction between production and market equilibrium easily reasserts itself.

But if one considers a city in which many origin-destination pairs have access to a common network of streets, the situation is complicated by the fact that there are as many distinct commodities as there are distinct origin-destination pairs. One is in effect dealing with a great number of interrelated production and market equilibria without being able to associate a unique cost function[2] to each commodity. This indeterminacy arises because, in a congested network, input requirements for the production of a given number of trips between a particular pair of points depend on the output levels already occurring among other pairs and influence input requirements for those output levels. In consequence, unless the pairs are arbitrarily

[2] As pointed out by Michael Florian using classical language, although one cannot associate a cost function to each commodity, one can still in equilibrium —characterized by flows that are unique but routes that are not— impute “externality costs” (to use classical language) to each commodity.

ordered, there is no unique way of imputing the technological externalities – and resulting increased production costs - to specific processes.

The difficulty of specifying interrelated production processes should not deter us from concluding that delay curves are never in practice good proxies for supply curves and that estimates of delay curves yield a particular projection of the production function. Let us add that observations on which such estimates are based probably do not represent market equilibria and that the task of finding the extent to which they do will require the formulation and estimation of market supply curves - a thoroughly underdeveloped area of transportation research.

References

Potts, Renfrey B. and Oliver, Robert M., Flows in Transportation Networks, Academic Press, New York, 1972, p. 50.

Baumol, William J., Economic Theory and Operations Analysis, Prentice-Hall, Second Edition, 1965, ch. 14.