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PREFACE

The present report is based on a case study of the Swedish railways, which have been used for two different work packages, wp6, which deals with the supplier's marginal cost, and wp7 which deals with the user costs and the implication of the "Mohring effect" for the price-relevant marginal cost. These two aspects of the cost structure are quite interrelated, which means that the model applied for the cost analysis should treat both aspects together: this has been done in the present case. The same model as was used in the contribution to wp6 is presented here, too, in a shortened version. Here the main discussion is focused on the Mohring effect, its definition, measurement and importance for the price-relevant marginal cost in the case of interurban rail transport.

SUMMARY

The ”Mohring effect” is fairly well understood, and known so far as its relative size is concerned in urban public transport, where it mainly manifests itself in the form of shorter waiting times at stops and stations when the frequency of service is increased. In the present case of interurban rail passenger transport, it is less easily measurable, because the main waiting time does usually not take place at the station. When the headway (= time interval between departures) exceeds ten to fifteen minutes, the travellers normally inform themselves of departure times in advance, and arrive just a few minutes before the train departure, irrespective of the headway. The main user cost occurs at the destination, where the arrival will be more or less early, or sometimes too late. In addition, the return trip may also have to be made at an inconvenient time, because the end of the activity at the destinations, which is the purpose of the trip, is not adjusted to train departure times. The inconvenience caused by infrequent train services is often referred to as “disguised waiting time”. This “time” cost could be assumed to be proportional to the headway, just as actual waiting time at stops of frequent urban public transport services can be assumed to be equal to half the headway, on average, and the Mohring effect could be calculated by finding out the unit value of disguised waiting time.

We prefer to call the user cost in question “headway cost”, because it can manifest itself in varied ways, and it cannot be taken for granted that it is proportional to the time interval between departures.

The purpose of the present case study is to derive the “behavioural” headway cost on the part of the supplier of train services, implied by the pattern of the input of engines, and carriages by the Swedish Railways (SJ) on different rail lines served by flexible-formation trains. Presumably rail service managers should know how passengers value frequency of service relative to the fare, and allocate engines between different lines in accordance with this calculation and the costs of engines including their drivers. In any case, it could be argued that the calculation of the price-relevant marginal cost of rail transport should be based on the actual supplier behaviour, rather than on a hypothetical “optimal” adjustment of train services.

A cross-section study of the number of train departures per day and the total number of passengers per day on 32 lines of widely different distances in the Swedish rail network reveals that, if frequency of service is plotted against the square root of the number of passengers per

line kilometer, a linear function with the origin as starting-point fits the observations extremely well: the adjusted R^2 = . 955 and the t-value = 27. A railway line optimization model shows that such a “square root law” applies in case the headway cost per trip is proportional to the headway, i.e. constant per unit of the time interval between train departures.

Therefore, the first conclusion is that the Swedish Railways managers responsible for the flexible formation train services believe that the headway cost function takes this shape. As to the unit value, it can be noted that the “square root law”, i.e. proportionality between frequency of service, and the square root of the density of demand, applies irrespective of the actual value of the headway cost per unit of time, as long as a linear shape can be assumed. However, from the cross-section study it is possible to find out the implicit unit value, too. By comparing the proportionality constant found in the regression analysis with the result of the optimization model, it is concluded that railway managers behave as if they believe the headway cost per hour to be about *2 Euro*, on average. (This figure is to be interpreted as an average of private and business trip values).

This value can be compared to the values used by Railtrack of Sweden in order to take the Mohring effect into account in Cost/Benefit-Analysis of rail track investments. The latter values are obtained from stated preference studies of rail passengers’ hypothetical choices, and range from less than 1 Euro to more than 10 Euro per hour. Most of the span is explained by the great difference between private and business trip values. In the second place, there is a basic difference as to the shape of the headway cost function assumed. The function assumed in Railtrack CBA is tapering off with increases in the headway: for example, below half an hour the value is 2 Euro per hour (that is 2/60 Euro per minute) for private trips, and above 2 hours’ headway, the value is only . 6 Euro per hour. A rough average value, including both private and business trips on different lines with widely different demand density, would be closer to 3 Euro than to 2 Euro per hour, so it seems that the belief of SJ is more conservative as regards the value to passengers of higher frequency of service than the belief of Banverket (the Swedish Railtrack).

1. INTRODUCTION

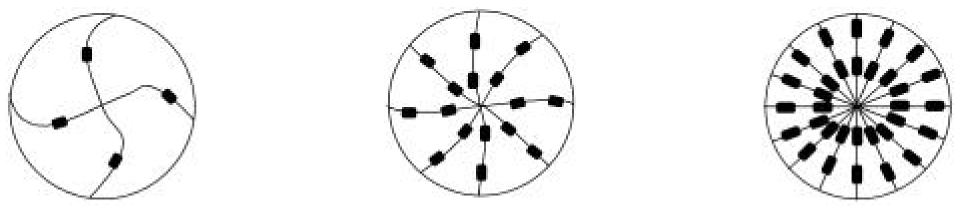
1.1 Problem, purpose and plan of the study

Jointness of capacity and quality is a common feature in transport, both for transport infrastructure, and schedule public transport (SPT). In the former case this results in relatively low levels of price-relevant marginal costs in non-urban road (Walters 1968) and rail networks, as well as in regional airports and seaports. In the case of SPT the Mohring effect also tends to make the price-relevant marginal cost fall well below the service operator's (supplier's) marginal cost, unless it is quality-of-service-compensated. In this report the question is: how important is the jointness of capacity and quality in interurban rail passenger transport? Will the Mohring effect, which is fairly well studied in urban bus transport, also make a substantial difference for optimal pricing of medium- to long-distance passenger transport by rail? These questions are addressed by a case study of the Swedish railways.

Before that the "generalized Mohring effect" is briefly explained, and its importance in urban transport by road and rail is illustrated. Some empirical material concerning frequent services, where a large proportion of passengers arrive to stops and station without knowing the exact departure time, partly because they do not bother to consult time-tables, partly because buses and train are often late (or early). Then the methodological problems of headway cost estimation are discussed for situations, where the headway is wide enough for practically everybody to think it best to inform themselves in advance about departure times (in section 2). After that the chosen approach in this study to estimating the Mohring effect is outlined. It combines an optimization model for a railway line (section 3) and a cross-section analysis of rolling stock input and trip output of some 30 different lines in the Swedish rail network (section 4). The optimality conditions derived from the model are used for interpreting the empirical findings in order to derive a value of the headway cost per hour (section 5). Finally the result is compared to previous findings and remaining research problems are briefly discussed.

1.2 The generalized Mohring effect

The “Mohring effect” has its name from a pioneering article by Herbert Mohring in 1972, “Optimization and scale economies in urban bus transportation”. As the title suggest, Mohring pointed out that significant economies of scale are prevailing in urban bus transport which has profound effects both on investment and pricing policy. It is not buses, which are special, but scheduled public transport (SPT) irrespective of which type of vehicles, defined from a technical point of view, are used. Since SPT-services are confined to specific routes and departure times, there are costs of access to the services which have both a time and a space dimension. The “generalized Mohring effect” is defined in a spatial context like that illustrated in fig 1, and results in very pronounced economies of demand density.



(1) four small buses ***(2) Sixteen mediumsize buses*** ***(3) Sixty-four big buses***

Figure 1 **Buses and bus lines in Circletown for three different levels of the density of demand**

In an urban bus transport system, for example, the economies of density of demand takes a number of expressions:

- more bus lines means less access time
- when bus lines are denser, each line can be straighter to save travel time
- more buses on each line means less waiting-time at bus stops
- buses should be successively bigger as the density of demand is increasing, which will reduce the bus operator's cost per trip.
- bigger buses can have a higher rate of occupancy with impunity as to queuing time for passengers.

1.3 The simple case of frequent urban bus and train services

In the short run, the most easily changeable quality of service is the frequency of service, so the most common demonstration of what the Mohring effect implies for optimal pricing of urban bus transport is as follows:

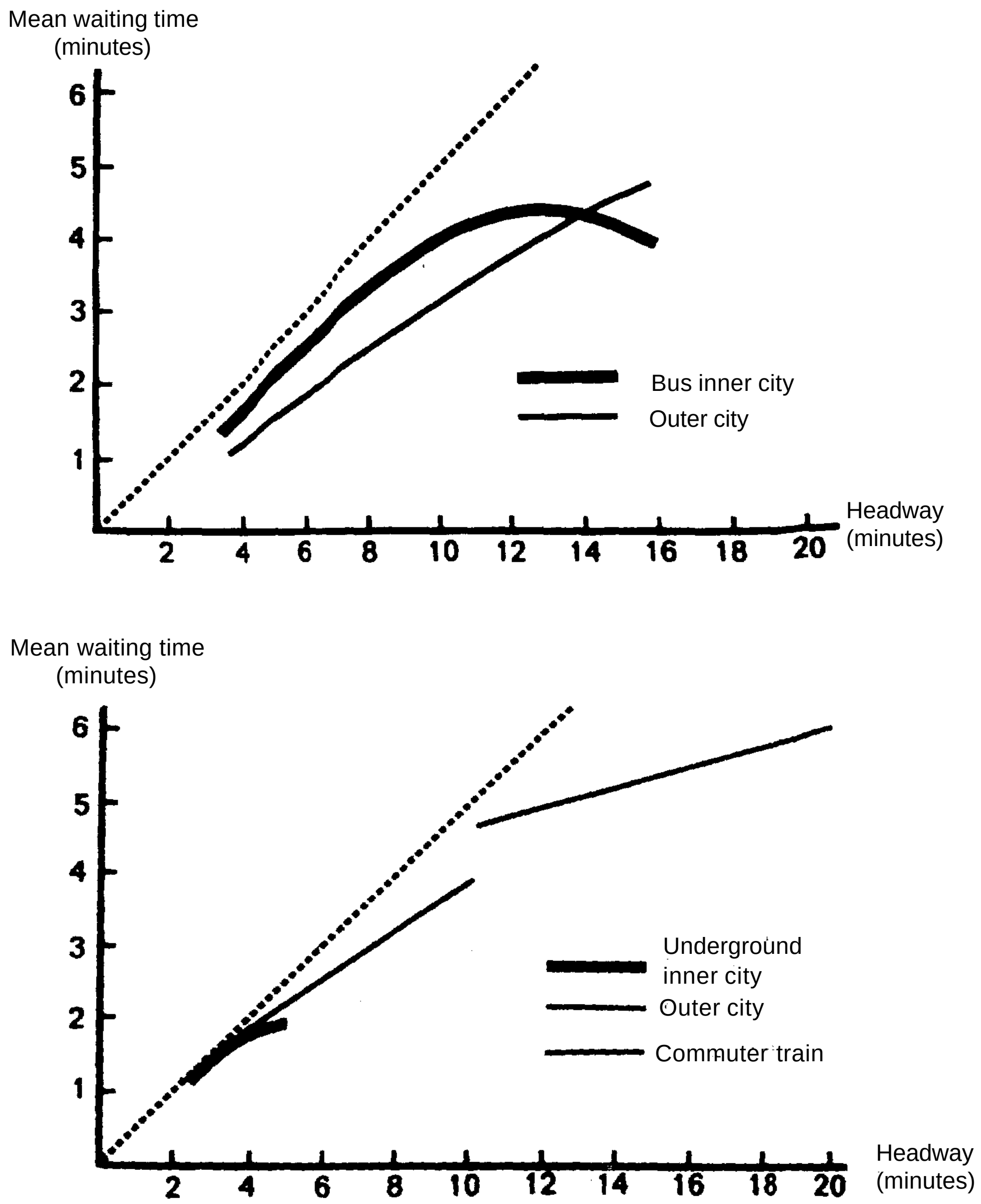
$$MC = \frac{\Delta C}{\Delta B} - \frac{vB\Delta t}{\Delta B}$$

ΔC = incremental cost of the bus company for putting in another bus in operation
B = existing number of passengers
ΔB = number of new passengers carried by the additional bus
Δt = waiting-time saving per trip by existing passenger
v = value of one minute waiting-time saving

If $\Delta C/\Delta B$ is assumed to be equal to the bus operator's average cost, and mean waiting time at bus stops is taken to be equal to half the headway (= time distance between busses), and if the waiting time cost is assumed to be twice the riding time cost of passengers, it follows that MC is only about 50% of the bus operator's average cost; on thin routes this percentage is less, and on dense routes it is somewhat higher.

When it comes to rail passenger transport, the same simple MC-formula can be used so far as frequent commuter train services in cities are concerned. The Mohring effect, i.e. the last, negative term of the formula above would under normal conditions be of a similar order of magnitude, both absolutely, and relative to the operator's average cost.

The attractive simplicity of this special urban case is not a general characteristic. Mean waiting time at stops of scheduled public transport equal to half the headway is a convenient assumption, but just an approximation, which will be less and less accurate as the headway is increasing. The relationships between mean waiting time and headway of the diagrams in fig 2 is an illustration of the basic fact that travellers increasingly take the trouble to consult a time-table in advance when headways exceed ten minutes. They do no longer arrive at stops and stations completely at random, but a larger and larger proportion of the travellers adjust their arrival time to a couple of minutes (safety margin) before scheduled departure time.



Source: TU:71: Trafikundersökningar i Stockholmsregionen hösten 1971, resultatrapport nr 1.
Stockholms Läns Landsting, Trafiknämnden

Figure 2 **Mean waiting time for buses and trains in Stockholm**

That there is a sacrifice involved in getting and assimilating the time-table information is clear from the fact that the travellers trade in expected waiting time for not having to obtain the information. This trade-off ceases fairly soon after the headway has passed a quarter of an hour. However, this does, of course, not mean that the user cost will stop rising with further increases in the headway.

2. METHODOLOGY

When it comes to interurban rail transport, the frequency of service is seldom high enough for passengers to go to the station without consulting a time-table beforehand. The diagram of fig 3 below illustrates the basic hypothesis as regards the shape of the user cost of frequency delay as a function of headway. From now on this user cost is called *headway cost*, and the headway cost per trip is denoted **h**.

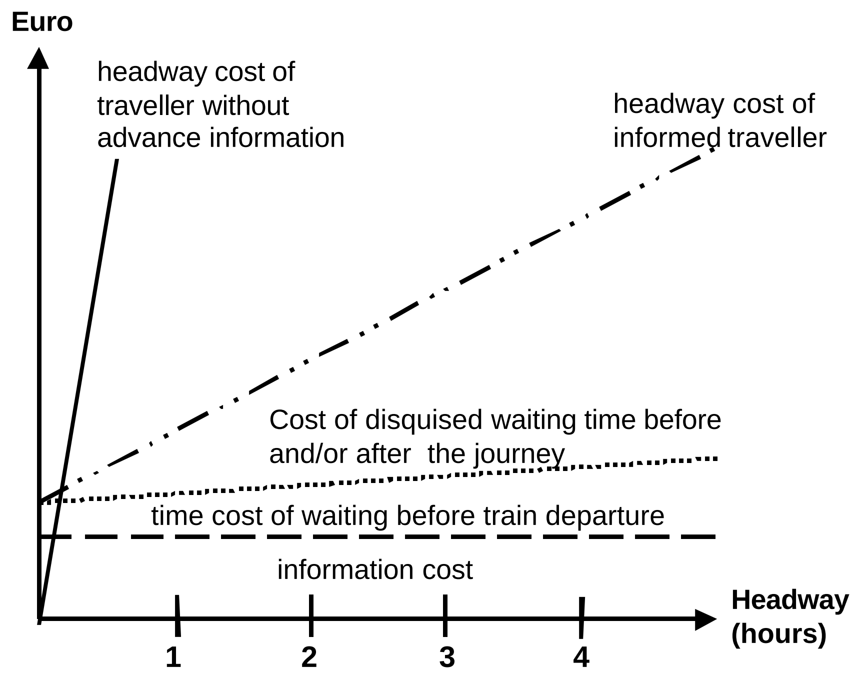


Figure 3 **The composition of the headway cost**

The longer the headway is, the more dominant the "disguised waiting time" component will be. Unfortunately, it is the headway cost component which is most difficult to quantify and monetize. There are three main methods to calculate the headway cost in the present case of well informed travellers.

1) Stated preference studies of travellers' willingness to pay for reduced headway.
2) Revealed preference studies of the same, that is travel demand estimation based on actual travel data, where variations in the frequency of service are contained.
3) Studies of SPT-operators trade-off between lower traffic operation costs obtained by running larger vehicles, and the higher headway costs for their passengers.

The big problem of (2) is the direction of causality: a high correlation is normally found between the volume of travel and the number of vehicles (buses, trains, airplanes) plying a particular route, and the question is, if it is the high frequency of service that causes the high volume of traffic, or the other way round? Certainly, by far the strongest effect is from travel volume to number of vehicles: between pairs of big cities there are much higher SPT-service frequencies than between pairs of smaller towns. The high frequency of service will itself induce additional travel, but which proportion of the total travel is induced (i.e. would not be there in a situation of a reduced frequency of service) is quite tricky to detect.

Results of stated preferences studies of headway costs have been used in Sweden since 1995 in the Cost-Benefit Analyses of rail investments carried out by the Swedish Railtrack. Before that much higher values were used; the common practice was simply to transfer headway cost values from local public transport, which represented actual waiting time at the bus stops, and metro or commuter train stations, to interurban train services, where travellers are using time-tables, As seen in table 1a below the headway cost per hour is nowadays considered to be many times lower in a situation, where the headway is so substantial that travellers take the trouble to inform themselves about departure times.

Table 1a **Values of headway reduction for private trips according to the Swedish CBA manual used for transport investment planning, Euro per hour**

HEADWAY RANGE, minutes	**Short-distance trip (< 50 km)**	**Long-distance trip (> 50 km)**
< 10 min	6.5	3
11-30 min	2.0	3
31-60 min	1.8	3
61-120 min	1.1	1.5
> 120 min	0.6	0.7

Source: SIKA (1999)

Table 1b **Values of headway reduction for business trips according to the Swedish CBA manual used for transport investment planning, Euro per hour**

HEADWAY RANGE, minutes	**Train**	**Air**
< 60 min	10.8	12.9
61-120 min	7.5	10.8
> 120 min	6.5	8.6

Source: SIKA (1999)

The stated preference studies have weaknesses but the ongoing methodological development will improve the practice. For example, in the first Swedish applications the practice was to ask travellers what additional payment they would be willing to make in case the particular train they are on would leave 10 minutes earlier. However, this does not seem completely relevant: many travellers have adjusted to the present time-table, and may have difficulties to image what they could make of being 10 minutes earlier at the trip destination. Nowadays the practice is to confront travellers with complete time-tables, and ask for their willingness to pay for an improved frequency of service.

The pattern of strongly falling headway costs per minute with headway increases, particularly pronounced for short-distance private travel (se table 1a), is a bit puzzling. It means, for example that the headway cost of passengers of a bus service with only one bus per hour is no more than twice as high as the headway cost per trip by a bus line with a frequency of service of 12 buses per hour.

There are obviously still unsolved problems at least at a more detailed level, and it is difficult to say whether revealed or stated preferences studies can be further refined to settle the remaining issues. Continued research in both fields is called for.

2.1 The chosen approach

In the meantime a third method of estimating the headway cost is worth exploring. This is to study the trade-off that train operators make between increased costs for themselves of higher frequency of service, and lower headway costs for their passengers.

For this purpose a cost model of a railway line is developed, where both producer and user costs are included. All factor prices are known to us except the headway cost, h(F) per trip, where F is frequency of service. The idea is to minimize the total costs and solve for **h** from the minimum cost condition. It is important to note that both a profit-maximizing and a social surplus-maximizing train operator would aim at minimizing the sum of total producer and user costs *for each level of output*. In both cases this is a necessary (but, of course, not a sufficient) condition for optimization. The distinguishing feature is that the profit-maximizer will produce a smaller output than the social surplus-maximizer. It is difficult to say whether SJ was a profit-maximizer or a social surplus-maximizer in the middle of the 1990s, but that does consequently not matter for this approach to the headway cost estimation.

The next step is to produce a cost model for the mimization exercise. The model presented below was used also in the UNITE wp6 report (Ericsson and Jansson, 2001). Here the presentation is brief, and the whole optimization procedure is not shown.

3. BASIC MODEL OF SEPARATE DOUBLE-TRACK RAILWAY LINE

The symbols used in the basic model can be categorized as (i) line characteristics, (ii) demand and supply quantities, (iii) factor costs and prices and (iv) quality of service or passenger cost parameters.

(i) **line characteristics**

D = distance (one round trip)

(ii) **demand and supply quantities**

Q = g(GC) = transport demand in terms of number of trips as a function of the generalized cost, GC

γ = average trip length

M = number of locomotive engines

N = total number of carriages

L = train length = number of carriages per train = N/M

k = number of round voyages per traffic day made by locomotive engines

H = service hours per day (=”traffic day”)

F = number of departures per traffic day (=frequency of service); F = kM

n = number of seats per carriage

Y = transport supply in terms of “seat-rounds” = nLF

(iii) **Factor costs and prices**

a = time cost of engines including engine-driver’s wage cost

b = time cost of carriages including guards’ wage costs

c = $\frac{c_1}{L} + c_2$ = distance-dependent cost per carriage-km

c_1 = “train-kilometre cost” = f(L,V)

c_2 = “carriage-kilometre cost”

(iv) **Quality and passenger cost parameters**

V = running speed

T = total round voyage time

t = total station time per round trip

t_0 = fixed station time including time buffer

t_1 = variable station time

ϕ = occupancy rate

v = $v(\phi)$ = time value per hour of travel on the train

h = h(F) = frequency delay or "headway cost" per trip

w = headway cost per hour

The scheduled time in the timetable for one round trip includes two main components: running time and station time including retardation and acceleration time. Running time is given by the ratio of distance D and running speed V, which is assumed to be given in the model. The station time can be assumed to consist of two components: one constant, and one trip-volume dependent component. The latter component should, in principle, be dependent both on the number of passengers boarding/alighting each carriage, which depends on the occupancy rate, and on the number of carriages (given the occupancy rate).

$$T = \frac{D}{V} + t_0 + t_1(f, L) \tag{1}$$

Railway people claim that t_0 is by far the dominant component of station time. Given the occupancy rate the timetable is adjusted to the number of stops made at stations rather than the expected number of passengers. The amount of time allowed for boarding/alighting is fixed regardless of the train length and expected number of passengers boarding/alighting at each station (passenger inlets and outlets are proportional to train length). The time added to t_0 is mainly considered as a time buffer to avoid delayed departures. This means that all three components of T are given in the model.

The total cost of the transport producer depends on the inputs of engines, carriages, staff and energy. The energy consumption depends on train weight, here represented by train length L, speed V, and line distance D. Staff inputs are in fixed proportion to the number of engines, and carriages, respectively.

It is possible to view the producer’s total cost as consisting of four components, which are proportional in turn to (1) the number of engines, (2) the number of carriages, (3) the number of train-km, and (4) the number of carriage-km.

$$TC_{prod} = aM + bN + c_1 FD + c_2 kND \qquad (2)$$

Besides the price of a ticket the passengers' generalized cost, *GC* includes various time costs. The cost of travel time on the train depends i.a. on the crowding, represented by the occupancy rate, f . Of particular interest for our problem is the “frequency delay” or “headway cost”, *h(F)*, which can be described as “the necessity to adapt to more or less infrequent departures”. As before we write *GC = P + Z*, where *Z* is the real part of the generalized cost. The total system cost is written:

$$TC = TC_{prod} + h(F) \cdot Q + v(f) T f n L F \qquad (3)$$

3.1 *Conditions for social surplus maximization*

For expository reason an expression for the social surplus is formed, which is to be maximized in order to derive the price-relevant marginal cost. This was done in the report to wp6 (Ericsson and Jansson, 2001). Here just the cost minimization condition with respect to the number of locomotive engines, **M** is derived.

The relevant lagrangian expression including the demand = supply equilibrium condition is written like this:

$$\Pi = P \cdot g(GC) + \int_{GC}^{GC^*} g(GC) \cdot dGC - TC_{prod} - l\left[g(GC)\frac{g}{D} - fnLF \right] \qquad (4)$$

Factors influencing the capacity and quality of service include the number of engines, and the frequency of service *F*, the total number of carriages, *N* and the train length, *L*. The optimality conditions are obtained by setting the derivatives of P with respect to *P* and the mentioned factors of production equal to zero.

First comes the pricing conditions, which gives us the optimal price in terms of the lagrangian multiplier, l

$$\frac{\partial \Pi}{\partial P} = P\frac{\partial g}{\partial GC} + g(GC) - g(GC) - l\,\frac{g}{D}\cdot\frac{\partial g}{\partial GC} = 0 \qquad (5a)$$

$$P = l\,\frac{g}{D} \qquad (5b)$$

Setting the derivative of Π with respect to the number of engines, M equal to zero is a necessary least-cost condition for each level of output.

$$\frac{\partial \Pi}{\partial M} = P\frac{\partial g}{\partial GC}\cdot\frac{\partial h}{\partial F}\cdot k - Q\frac{\partial h}{\partial F}\cdot k - a + \frac{\partial f}{\partial L}\cdot\frac{N}{M^2}kMD - c_1 kD - l\,\frac{g}{D}\cdot\frac{\partial g}{\partial GC}\cdot\frac{\partial h}{\partial F}\cdot k = 0 \qquad (6)$$

Eliminating P by using (5b), the least-cost condition is reduced to:

$$-Q\cdot\frac{\partial h}{\partial F} - \frac{a}{k} + \frac{\partial f}{\partial L}LD - c_1 D = 0 \qquad (7)$$

This gives us the factor representing the "Mohring effect", $Q\frac{dh}{\partial F}$ in terms of certain producer costs, which can be further reduced, using some empirical results reported in Ericsson and Jansson (2001).

There it is shown that $c_1 = f(L, V)$, i.e. train energy consumption as a function of the number of carriages (L) and speed (V) is proportional to L, for each given speed, This means that the elasticity of c_1 with respect to L is unity. Rewriting (7), and using this result gives:

$$Q\frac{\partial h}{\partial F} = \frac{a}{k} + c_1 D\left(\frac{\partial f}{\partial L}\frac{L}{f} - 1\right) = \frac{a}{k} \qquad (8)$$

The result is thus simply that cost-minimization requires that, on the margin, the incremental cost of another engine (including the driver) per round voyage is equal to the total headway cost reduction for existing passengers obtained by a unit increase in the frequency of service. This puts the light on the double role of locomotive engines, which is good to bear in mind: on one hand, engines are obviously necessary inputs for train transport *capacity*. On the other hand, given the total number of carriages employed on a particular line, another engine would not increase the total capacity, but it would increase the *quality* of service by the possibility to distribute the given number of carriages between an increased number of trains. This jointness of capacity and quality has a profound effect on the optimal price level, which was thoroughly discussed in Ericsson and Jansson (2001). There it was pointed out, among other things, that strangely enough the *shape* of the headway cost function, h(F), which determines the Mohring effect, is inconsequential for optimal pricing. The financial result of optimal pricing is anyway that the engine and engine-driver cost would remain uncovered.

For pricing purposes there is thus no urgent need to explore the function h(F) further, but for other purposes it can certainly be interesting to know more about this function. To that end, we have studied the behaviour of SJ as regards the frequency of service on different lines of different length, and widely different numbers of passengers.

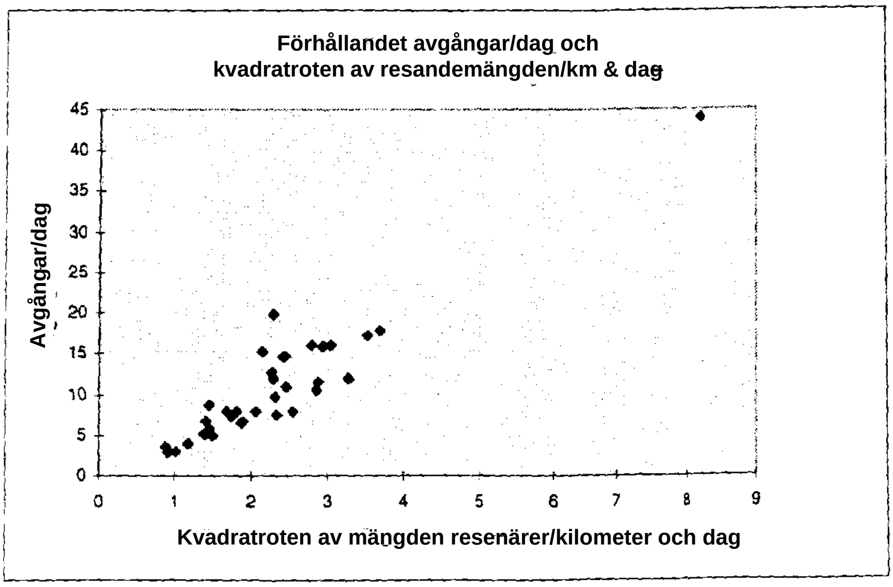
4. CROSS-SECTION REGRESSION ANALYSIS OF SERVICE FREQUENCY AND DENSITY OF DEMAND

The estimation problem is that the condition Q $\partial h/\partial F = a/k$ is not sufficient for determining the shape of the function h(F). To come further, it is necessary to look for a pattern of factor inputs and output of rail passenger transport, which could give us a clue about what SJ believes is its customers' valuation of the quality of frequency of service.

To be able to estimate by regression analysis a rail transport production function, cross-section data has to be relied on. Over time too little of factor combination changes happen. It is true that in the 1990s a new system of high-speed trains was introduced in the Swedish railway network, so today we have two parallell inter-city passenger services on some railway lines: fixed formation high-speed trains and flexible-formation "ordinary" inter-city trains. This is, of course, a very interesting technical and commercial development, but a single discrete jump from one technology to another does not make a sufficient data source for statistical curve fitting. On the other hand, taking each more or less separate railway line during a limited period of time as the observations – there are about 30 such useful observations of the Swedish rail transport system – there is at least one striking difference in the set of inputs between different lines, which could be significant factors in a regression analysis: the number of carriages and engines on individual lines differs greatly, and particularly interesting is that the ratio of the number of carriages to the number of engines, i.e. the average train length is also very different on different lines.

We have worked in this field for a number of years [Ericsson (1997), Molinder (1998), Persson (1999)], and for the UNITE-project the idea was to examine how the Swedish railways takes the Mohring effect into account in the train formation and scheduling. Comparing different lines with widely different volumes of travel, the question was, what the trade-off between the costs of additional engines and the frequency delays of passengers looked like.

A plot of frequency of service versus the square root of the number of (boarding) passengers per kilometer on the different lines is given in fig 4 below. As seen a linear function with the origin as starting-point is certainly suggestive. The goodness of fit is high : adjusted R square is equal to . 955 and the t-value is 27.



Source: Persson (1999)

Figure 4 **Frequency of service versus the square root of passenger trips per kilometer in the Swedish rail network**

The reason why the correlation between F and $\sqrt{Q/D}$ was tried out is obvious. The simpliest notion about the headway cost per trip is that, like in frequent urban bus transport, it would be proportional to the headway, i.e. constant per unit of time, but presumably lower per hour when it takes the form of disguised waiting time rather than actual waiting time at a bus stop. In case the headway cost per hour (w) is constant, cost minimization would lead to proportionality between frequency of service and density of demand, which is easily checked with reference to (8) above:

Setting h(F) = wH/F,

where w = headway cost per hour, and H = total service hours per day, it follows that

$$Q\frac{\partial h}{\partial F} = -Q\frac{wH}{F^2} = \frac{a}{k} \tag{9}$$

which implies proportionality between frequency of service, and the square route of the density of demand.

5. DERIVATION OF THE HEADWAY COST PER HOUR

Accepting this result so far as the functional form of the headway cost function h(F) is concerned, the next question is about the value of w. The gradient of the linear relationship between frequency and the square root of demand density tells us something about that. A steeply sloping line indicates a relatively high value of w, resulting in a high frequency of service for each particular level of demand. The slope is also critically dependent on the units of measurement. In the regression analysis the proportionality constant took the value of 4.924 when density of demand was measured by the number of boarding passengers per kilometer and day, and the frequency of service by the number of train departures between 6 a.m. and 8 p.m per day. From the cost-minimization condition it is possible to get a value for w, but we have to change over from the assumed integer k, standing for the number of round voyages performed by each train per day on a particular line, to the corresponding continuous variable VH/D.

With the above specifications the cost minimization condition (8) and (9) takes the following shape:

$$Q\frac{wH}{F^2}=\frac{aD}{VH} \tag{10}$$

From this equation, frequency of service, F can be expressed as a linear function of the square root of the density of demand, Q/D:

$$F=H\sqrt{\frac{wV}{a}}\sqrt{\frac{Q}{D}} \tag{11}$$

The first two factors of (11) correspond to the proportionality constant 4.924 of the linear regression equation fitting the observations plotted in fig 4 above:

$$H\sqrt{\frac{wV}{a}}=4{,}924 \tag{12}$$

Daily service hours, H is set = 14 and the average speed, V (the inverse of T) is assumed to be 90 km/h. From the CBA manual of Swedish Railtrack (Banverket, 2000) the engine cost per day is calculated (a = 13 500 SEK, which is close to the value obtained in Molinder, 1998 in a similar study), which finally gives the value of the headway cost per hour, w.

w = 2 Euro per hour

6. DISCUSSION OF THE RESULT AND REMAING PROBLEMS

Compared to the “official” values cited in table 1a above for long-distance private trips based on stated preference studies of travellers’ choice, this value falls in between the values applicable to the headway range less than one hour, and the values in the headway range above one hour. It can also be noted that the values of headway reduction for business trips in table 1b are many times higher than the value derived here. This is, of course, to be expected. Just like values of travel time, headway cost values should be much higher for business trips than for private trips. The present value, 2 Euro/h should be viewed as an average value of medium- and long-distance trips by private as well as business travellers. A weighted average of all relevant values of table 1a, and 1b would be closer to 3 Euro than to 2 Euro, so one interpretation of the result can be that the revealed behaviour of SJ as regards the trade-off between frequency of service and costs of additional engines is too cautious. SJ should spend more on improving the frequency of service.

Another interpretation can be that it is the experienced operator, who has first-hand knowledge of the market, who is right, rather than the academic researchers playing stated preference games. Anyway, there are more research to be done. In the present study of operator behaviour it was concluded that the value per hour of headway reduction is constant, independent of the prevailing headway, whereas stated preference studies indicate a falling value per hour as the headway is increasing. The utility-theoretical foundation of this feature is weak. Why would the disadvantages of more and more infrequent train services be tapering off? The inconvenience should instead be accelerating. On the other hand, it can be argued that if the frequency of service is being successively reduced, anyone who is concerned about frequency has already chosen another mode, and in the end the few remaining passengers are people who do not care, i.e. have a low valuation of service frequency.

Revealed preference studies of travellers’ behaviour – method 2 (See page 7) – is another line of research which should be further developed.

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