MODELLING TRAFFIC SIGNAL CONTROL AND ROUTE CHOICE

traffic signal control, route choice and day-to-day dynamics

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introduction

aim

macrosimulation within-day static strategic planning

effective
(consistency)

efficient
(optimality)

successful
(convergence)

robust solution
introduction

contents

general framework of

• problems and modelling approaches

addressing

• solution existence
• solution uniqueness
• convergence (stability)
• large scale applications

highlighting

• established results
• open issues
introduction
overview

- review of basic
  - signal setting
  - network level-of-service
  - route choice

- signal setting and equilibrium
  - equilibrium assignment
  - global optimization
  - heuristic approaches

- signal setting and day-to-day dynamics
  - day-to-day dynamic assignment - stability conditions
  - static optimization with stability constraints
  - dynamic optimization
signal setting with given flows

- decision variables: given
- signal timings \( g \)
- flows \( f \)

**objective functions** \( \varphi(g, f) \):
- min total delay
- min total travel time & delay
- max capacity factor
- min pollution, ...

arrival flow based methods or cyclic controllers
(no traffic-responsive)
signal setting optimization with given flows

parametric optimization w.r.t. timings for given flows $f$

$g^{ss} = \arg\min_{g \in S_g} \varphi(g, f)$

$S_g$ signal timing feasible set

$g = g^{ss}(f) \in S_g$

existence – uniq. for convex o.f. l.s. applications
signal setting with feasible flows

decision variables:

- signal timings $g$
- flows $f$

objective functions $\varphi(g, f)$:
- min total travel time
- min total delay
- max capacity factor
signal setting optimization with feasible flows

**optimization**

w.r.t. timings and flows

\[ g^{USS}, f^{USS} = \arg \min_{g \in S_g, f \in S_f} \varphi(g, f) \]

\[ S_f \text{ arc flow feasible set} \]

(uncostained flows)

\[ \varphi(g^{USS}, f^{USS}) \geq \varphi(g^{SS}, f^{SS}) \]

green

greater values

flow

existence – uniq. for convex o.f.
l.s. applications
assignment
modelling user route choice behaviour

- arc flow function

- flows

- costs
assignment
flow function

\[ f \in S_f \text{ arc flows} \]
\[ c \geq 0 \text{ arc costs} \]
\[ d \text{ demand flows} \]

arc flow function
\[ f = f(c; \theta, d) \in S_f \]
continuous, c. differentiable
monotone (for invariant RUM)
with sym. neg. semi-def. Jac.

\[ S_f \text{ arc flow feasible set} \]
non empty, compact, convex

\{ \text{network consistency equations} \}
\{ \text{route choice model (from RUM)} \}
assignment modelling congestion

- arc cost function

flows

signal timings

costs

traffic signal control, route choice, day-to-day dynamics
assignment
cost function

c ≥ 0 arc costs
f ≥ 0 arc flows
g ∈ Ŝg signal timings
green times, offsets, ...

arc cost function
c = c(g, f; μ) ≥ 0

continuous, c. differentiable
possibly monotone

Ŝg signal timing feasible set
non empty, compact, convex
assignment equilibrium

fixed-point models

flows

costs

EQUILIBRIUM

signal timings
assignment equilibrium

- parametric F-P for given g

- fixed-point model w.r.t. arc flows and costs
  \[ c^* = c(g, f^*; \mu) \]
  \[ f^* = f(c^*; \theta, d) \]

- fixed-point model w.r.t. arc flows
  \[ f^* = f(c(g, f^*; \mu); \theta, d) \]

existence – uniqueness
l.s. applications
signal setting with equilibrium flows

global optimization

flows

costs

EQUILIBRIUM

SIGNAL SETTING

signal timings
signal setting (global) optimization with equilibrium flows

- optimization w.r.t. timings and flows

\[
g^{\text{ESS}}, f^{\text{ESS}} = \arg\min_{g \in S_g, f \in S_f} \varphi(g, f)
\]

s.to equilibrium constraints

\[
f = f(c(g, f; \mu); \theta, d)
\]

\[
\varphi(g^{\text{USS}}, f^{\text{USS}}) \geq \varphi(g^{\text{ESS}}, f^{\text{ESS}})
\]

existence – uniqueness
l.s. applications
signal setting with equilibrium flows

heuristics: (external) recursive optimization
signal setting
with equilibrium flows

heuristics: (internal) embedded optimization

flows → SIGNAL SETTING → signal timings

costs
signal setting
heuristics consistent with equilibrium flows

- recursive optimization w.r.t. timings and flows
  \[ g^* = g_{SS}(f^*) \]
  \[ f^* = f(c(g, f^*; \mu); \theta, d) \]

- embedded optimization w.r.t. flows (equilibrium)
  \[ f^* = f(c(g_{SS}(f^*), f^*; \mu); \theta, d) \]

EQUIVALENT

existence – uniqueness
l.s. applications
signal setting with equilibrium flows: considerations

- signal setting with feasible flows \(^\circ\) gives better solutions than signal setting with equilibrium flows \(^\circ\)

- solutions of embedded and recursive optimization are equivalent

- global optimization \(^\circ\) for signal setting with equilibrium flows gives better solutions than embedded or recursive optimization

- uniqueness of global optimization \(^\circ\) does not guarantee uniqueness of embedded or recursive optimization
day-to-day dynamic assignment
modelling habit and inertia

flow (choice) updating equation

flows

costs

day t
day-to-day dynamic assignment flow (choice) updating equation

\[ f^t = \alpha \cdot f(x^t) + (1 - \alpha) \cdot f^{t-1} \]

- \( f^t \) arc flows at day \( t \)
- \( x^t \) user forecasted arc costs at day \( t \)

\( \alpha \in ]0,1] \)

fraction of users who reconsider yesterday choice

\( \alpha = 0.62 \)

\( \alpha = 0.42 \)

previous days

previous days
day-to-day dynamic assignment
modelling memory and learning

cost updating equation

flows

costs

day t

signal timings
day-to-day dynamic assignment
cost updating equation

$$x^t = \beta \cdot c(g^{t-1}, f^{t-1}) + (1 - \beta) \cdot x^{t-1}$$

$\beta \in ]0,1]$  
weight given to yesterday costs

$\beta = 0.62$  
$\beta = 0.42$  

previous days  
previous days
assignment
day-to-day dynamics

deterministic process models

traffic signal control, route choice, day-to-day dynamics

flows

costs

day t

signal timings

STABILITY CONDITIONS
assignment
day-to-day dynamics

for given timings \( g_0 \)

- deterministic process

models w.r.t. flows and costs

\[
\begin{align*}
x^t &= \beta \cdot c(g_0, f^{t-1}) + (1 - \beta) \cdot x^{t-1} \\
f^t &= \alpha \cdot f(x^t) + (1 - \alpha) \cdot f^{t-1}
\end{align*}
\]

with fixed-point states w.r.t arc flows and costs

\[
\begin{align*}
x^* &= c(g_0, f^*) \\
f^* &= f(x^*)
\end{align*}
\]

- stability conditions

\[\text{attractors}\]

- fixed-points (from \( \square \)) equivalent to equilibrium

\[\text{k-periodic (from } \square \text{)}\]

\[\text{quasi-periodic}\]

\[\text{a-periodic (fractal)}\]
deterministic process assignment
general fixed-point stability conditions

\[ \omega_a(\mu, \theta, d) : \] one of the n eigenvalues of
\[ J_c(f) \cdot J_f(c) = \text{Jac}[f(c, \theta, d)] \cdot \text{Jac}[c(f, g, \mu)] \]

\[ \omega_a^* \] one eigenvalue computed at a fixed-point state \((f^*, x^*)\)

**general stability condition**

\[ \omega_a^*(\mu, \theta, d) \in \text{SR}(\alpha, \beta) \quad \forall a \]

\[
\frac{(\text{Re}(\omega_a^* - 1) + e_R)^2}{e_R^2} + \frac{(\text{Im}(\omega_a^*))^2}{e_I^2} < 1 \quad \forall a
\]

\( \text{Re}(\bullet) \) the real part of the (possibly complex) argument,
\( \text{Im}(\bullet) \) the imaginary part of the argument

\[ e_R = \frac{(1 + (1 - \alpha)(1 - \beta))}{(\alpha \beta)} \geq 1 \]
\[ e_I = \frac{(1 - (1 - \alpha)(1 - \beta))}{(\alpha \beta)} \leq 1 \]
deterministic process assignment
special fixed-point stability conditions

\[ \omega_a = \text{Re}(\omega_a) \leq 0 \]
for \( J_c(f) \) sym. pos. def. \& \( J_f(c) \) sym. neg. semi-def.

special stability condition
\[ \text{MAX}_a | \omega_a^*(\mu, \theta, d) | < \omega^*(\alpha, \beta) \]

\[ \omega^*(\alpha, \beta) = (1 + 2 \cdot ((1 - \alpha) + (1 - \beta)) / (\alpha \beta)) \]

notes
several pairs \((\alpha, \beta)\) give the same \( \omega^*(\alpha, \beta) \)
\[ \omega^*(\alpha, \beta) = \omega^*(\beta, \alpha) \]
signal setting with stable equilibrium flows

static optimization

flows

costs

SIGNAL SETTING

signal timings

EQUILIBRIUM

STABILITY CONDITIONS
signal setting
static optimiz. with stable equilibrium flows

- static optimization w.r.t. timings and flows

\[ g_{SSS}, f_{SSS} = \arg\min_{g \in S_g, f \in S_f} \varphi(g, f) \]

s.to equilibrium constraints
\[ f = f(c(g, f; \mu); \theta, d) \]

stability constraints
\[ \omega_a^*(\mu, \theta, d) \in SR(\alpha, \beta) \quad \forall a \]

existence – uniqueness
stability – l.s. applications
signal setting
static optimiz. with stable equilibrium flows
approx. models for large scale applications

static optimization
w.r.t. timings and flows

\[ g^{sss}, f^{sss} = \arg\min_{g \in S_g, f \in S_f} \varphi(g, f) \]

s.to equilibrium constraints
\[ f = f(c(g, f; \mu); \theta, d) \]
approx. stability constraints
\[ || J_c(g, f) \cdot J_f(c) || < \omega(\alpha, \beta) \]

existence – uniqueness
stability – l.s. applications

assumptions:
- \( J_f(c) \) sym. neg. semi-def.
- \( J_c(f) \) symmetric pos. def.

- timings: green times
  (separable cost function)

solution approach:
evolutionary heuristics

extensions:
- green time scheduling
- coordination (offsets)
signal setting
static optimiz. with stable equilibrium flows

existence

depending on parameters such as $\theta$

stability constraint
non-active | active

solution existence
yes | no

$\theta = 0 \rightarrow$
deterministic

$\theta = 0 \rightarrow$
deterministic

$\theta = 24$

$\theta = 12$

$\theta = 18$

$\theta = 6$
signal setting with stable equilibrium flows

dynamic optimization

flows
SIGNAL SETTING
green splits
costs
day t
signal setting
dynamic optimiz. with stable equilibrium flows

- deterministic process models w.r.t. flows and costs and timings
  \[ g^t = \gamma \cdot g(f^{t-1}) + (1 - \gamma) \cdot g^{t-1} \]
  \[ x^t = \beta \cdot c(g^{t-1}, f^{t-1}) + (1 - \beta) \cdot x^{t-1} \]
  \[ f^t = \alpha \cdot f(x^t) + (1 - \alpha) \cdot f^{t-1} \]

- attractors
  - fixed-points
  - k-periodic
  - quasi-periodic
  - a-periodic (fractal)

with fixed-point states
w.r.t flows, costs, timings
\[ g^* = g(f^*) \]
\[ x^* = c(g^*, f^*) \]
\[ f^* = f(x^*) \]
signal setting

dynamic optimiz. with stable equilibrium flows

open issues

- choice of control policy $g(\bullet)$
  - aiming at stability
  - aiming at optimization
  - consistent with static optimization

- choice of parameter $\gamma$ (which affects stability)
  (note. fixed-points independent of $\gamma$)

- extension of stability conditions with
  stability region w.r.t. $\alpha, \beta$
signal setting with stable equilibrium flows: considerations

- Signal setting with equilibrium flows gives better solutions than signal setting with stable equilibrium flows:
  static optimization

- If timing strategy is $g_{SS}(\cdot)$:
  - Fixed-points of dynamic optimization and solutions of embedded and recursive optimization are equivalent
  - Signal setting with equilibrium flows gives better solutions than dynamic optimization

- Static vs. dynamic optimization: optimality & stability
conclusion
remarks

relevant issues

- equity $\rightarrow$ multi-criteria optimization
- local vs. global stability
- $\alpha, \beta$ depending on day $t$ towards stability
- capacity constraints

parameters related to user behaviour (ATIS) $\alpha, \beta$ and $\theta$
as well demand flows $d$ and capacity values
may greatly affect features of solution
such as optimality and stability (existence in same cases)
conclusion

research perspectives

- general approach to TSD
- possible extensions: analysis and design
  - *Urban Networks (including lane allocation)*
  - *Networks with ATIS*
  - *Parking control strategies*
  - *Tolling strategies*
  - *Transit operator strategies*
- multi-user classes
- large scale applications
conclusion

research perspectives

- more general deterministic process models
- global stability conditions from deterministic process models
- transient length
- calibration of updating parameters
- stochastic process models
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traffic signal control, route choice and day-to-day dynamics

Thanks.
Questions?
Comments!